OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

THURSDAY 18 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

Section A (36 marks)

1 Is the following statement true or false? Justify your answer.

$$x^2 = 4 \text{ if and only if } x = 2$$ [2]

2 **(i)** Find the roots of the quadratic equation $z^2 - 4z + 7 = 0$, simplifying your answers as far as possible. [4]

(ii) Represent these roots on an Argand diagram. [2]

3 The points A, B and C in the triangle in Fig. 3 are mapped to the points A′, B′ and C′ respectively under the transformation represented by the matrix $\mathbf{M} = \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$.

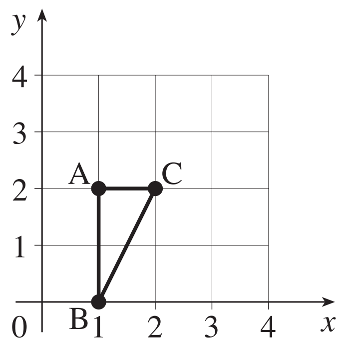


Fig. 3

(i) Draw a diagram showing the image of the triangle after the transformation, labelling the image of each point clearly. [4]

(ii) Describe fully the transformation represented by the matrix **M**. [3]

4 Use standard series formulae to find $\sum_{r=1}^{n} r(r^2 + 1)$, factorising your answer as far as possible. [6]

5 The roots of the cubic equation $2x^3 - 3x^2 + x - 4 = 0$ are α, β and γ.

Find the cubic equation whose roots are $2\alpha + 1$, $2\beta + 1$ and $2\gamma + 1$, expressing your answer in a form with integer coefficients. [7]

6 Prove by induction that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$. [8]

Section B (36 marks)

7 A curve has equation $y = \dfrac{5}{(x+2)(4-x)}$.

(i) Write down the value of y when $x = 0$. [1]

(ii) Write down the equations of the three asymptotes. [3]

(iii) Sketch the curve. [3]

(iv) Find the values of x for which $\dfrac{5}{(x+2)(4-x)} = 1$ and hence solve the inequality

$$\frac{5}{(x+2)(4-x)} < 1.$$ [5]

8 It is given that $m = -4 + 2\mathrm{j}$.

(i) Express $\dfrac{1}{m}$ in the form $a + b\mathrm{j}$. [2]

(ii) Express m in modulus-argument form. [4]

(iii) Represent the following loci on separate Argand diagrams.

(A) $\arg(z - m) = \dfrac{\pi}{4}$ [2]

(B) $0 < \arg(z - m) < \dfrac{\pi}{4}$ [3]

9 Matrices $\mathbf{M}$ and $\mathbf{N}$ are given by $\mathbf{M} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix}$.

(i) Find $\mathbf{M}^{-1}$ and $\mathbf{N}^{-1}$. [3]

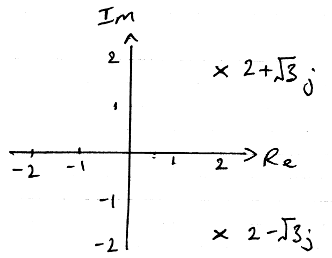
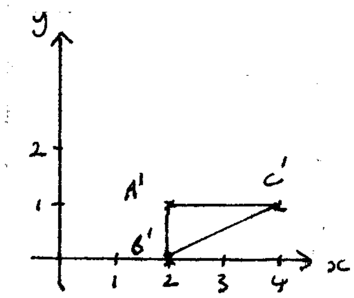
(ii) Find $\mathbf{MN}$ and $(\mathbf{MN})^{-1}$. Verify that $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$. [6]

(iii) The result $(\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ is true for any two 2×2, non-singular matrices $\mathbf{P}$ and $\mathbf{Q}$.

The first two lines of a proof of this general result are given below. Beginning with these two lines, complete the general proof.

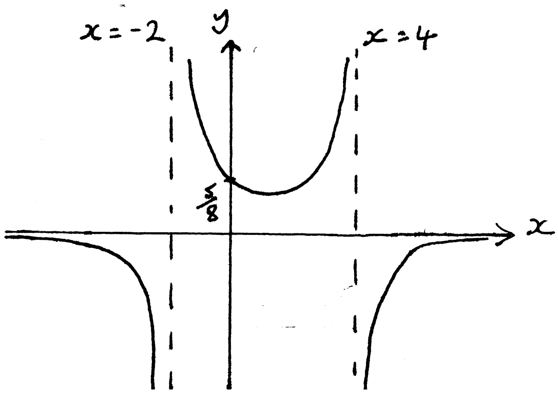
$$(\mathbf{PQ})^{-1}\mathbf{PQ} = \mathbf{I}$$
$$\Rightarrow\ (\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1} = \mathbf{IQ}^{-1}$$ [4]

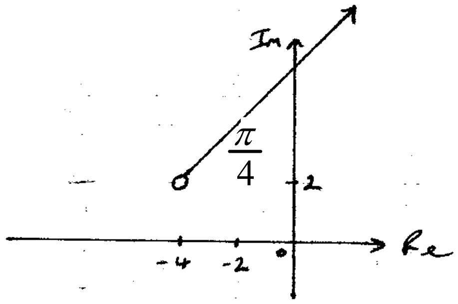
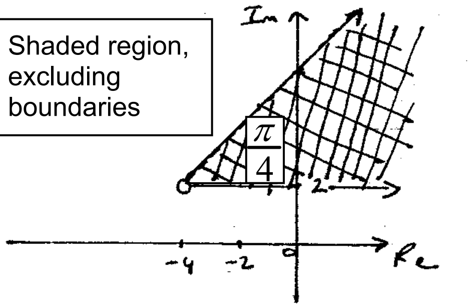
Mark Scheme 4755
January 2007

Qu	Answer	Mark	Comment
Section A			
1	The statement is false. The ‘if’ part is true, but the ‘only if’ is false since $x = -2$ also satisfies the equation.	M1 A1 **[2]**	‘False’, with attempted justification (may be implied) Correct justification
2(i)	$\frac{4 \pm \sqrt{16-28}}{2}$ $= \frac{4 \pm \sqrt{12}}{2}\text{j} = 2 \pm \sqrt{3}\text{j}$	M1 A1 A1 A1 **[4]**	Attempt to use quadratic formula or other valid method Correct Unsimplified form. Fully simplified form.
2(ii)		B1(ft) B1(ft) **[2]**	One correct, with correct labelling Other in correct relative position s.c. give B1 if both points consistent with (i) but no/incorrect labelling
3(i)	 $\begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 4 \\ 1 & 0 & 1 \end{pmatrix}$	B3 B1 ***ELSE*** M1 A1 **[4]**	Points correctly plotted Points correctly labelled Applying matrix to points Minus 1 each error
3(ii)	Stretch, factor 2 in *x*-direction, stretch factor half in *y*-direction.	B1 B1 B1 **[3]**	1 mark for stretch (withhold if rotation, reflection or translation mentioned incorrectly) 1 mark for each factor and direction

4	$\sum_{r=1}^{n} r\left(r^2+1\right)=\sum_{r=1}^{n} r^3+\sum_{r=1}^{n} r$ $=\frac{1}{4}n^2\left(n+1\right)^2+\frac{1}{2}n\left(n+1\right)$ $=\frac{1}{4}n\left(n+1\right)[n\left(n+1\right)+2]$ $=\frac{1}{4}n\left(n+1\right)\left(n^2+n+2\right)$	M1 M1 A1 M1 A1 A1 **[6]**	Separate into two sums (may be implied by later working) Use of standard results Correct Attempt to factorise (dependent on previous M marks) Factor of $n(n+1)$ c.a.o.
5	$\omega=2x+1\Rightarrow x=\frac{\omega-1}{2}$ $2\left(\frac{\omega-1}{2}\right)^3-3\left(\frac{\omega-1}{2}\right)^2+\left(\frac{\omega-1}{2}\right)-4=0$ $\Rightarrow\frac{1}{4}\left(\omega^3-3\omega^2+3\omega-1\right)-\frac{3}{4}\left(\omega^2-2\omega+1\right)$ $+\frac{1}{2}(\omega-1)-4=0$ $\Rightarrow\omega^3-6\omega^2+11\omega-22=0$	M1 A1 M1 A1(ft) A1(ft) A2 **[7]**	Attempt to give substitution Correct Substitute into cubic Cubic term Quadratic term Minus 1 each error (missing '= 0' is an error)
5	OR $\alpha+\beta+\gamma=\frac{3}{2}$ $\alpha\beta+\alpha\gamma+\beta\gamma=\frac{1}{2}$ $\alpha\beta\gamma=2$ Let new roots be *k*, *l*, *m* then $k+l+m=2\left(\alpha+\beta+\gamma\right)+3=6=\frac{-B}{A}$ $kl+km+lm=4\left(\alpha\beta+\alpha\gamma+\beta\gamma\right)+$ $4\left(\alpha+\beta+\gamma\right)+3=11=\frac{C}{A}$ $klm=8\alpha\beta\gamma+4\left(\alpha\beta+\beta\gamma+\beta\gamma\right)$ $+2\left(\alpha+\beta+\gamma\right)+1=22=\frac{-D}{A}$ $\Rightarrow\omega^3-6\omega^2+11\omega-22=0$	M1 A1 M1 M1 M1 A2 **[7]**	Attempt to find sums and products of roots All correct Use of sum of roots Use of sum of product of roots in pairs Use of product of roots Minus 1 each error (missing '= 0' is an error)

6	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ n = 1, LHS = RHS = 1 Assume true for $n = k$ Next term is $(k+1)^2$ Add to both sides $\text{RHS} = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ $= \frac{1}{6}(k+1)[k(2k+1)+6(k+1)]$ $= \frac{1}{6}(k+1)[2k^2+7k+6]$ $= \frac{1}{6}(k+1)(k+2)(2k+3)$ $= \frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$ But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for k = 1, it is true for k = 1, 2, 3 and so true for all positive integers.	B1 M1 B1 M1 M1 A1 E1 E1 **[8]**	 Assuming true for k. (k + 1)th term. Add to both sides Attempt to factorise Correct brackets required – also allow correct unfactorised form Showing this is the expression with $n = k + 1$ Only if both previous E marks awarded
			Section A Total: 36

Section B			
7(i)	$y = \frac{5}{8}$	B1 **[1]**	
7(ii)	$x = -2,\ x = 4,\ y = 0$	B1, B1 B1 **[3]**	
7(iii)	3 correct branches Correct, labelled asymptotes *y*-intercept labelled 	B1 B1 B1 **[3]**	Ft from (ii) Ft from (i)
7(iv)	$\frac{5}{(x+2)(4-x)} = 1$ $\Rightarrow 5 = (x+2)(4-x)$ $\Rightarrow 5 = -x^2 + 2x + 8$ $\Rightarrow x^2 - 2x - 3 = 0$ $\Rightarrow (x-3)(x+1) = 0$ $\Rightarrow x = 3 \text{ or } x = -1$ From graph: $x < -2$ or $-1 < x < 3$ or $x > 4$	M1 A1 B1 B1 B1 **[5]**	Or evidence of other valid method Both values Ft from previous A1 Penalise inclusive inequalities only once

8(i)	$\frac{1}{m}=\frac{1}{-4+2\mathrm{j}}=\frac{-4-2\mathrm{j}}{(-4+2\mathrm{j})(-4-2\mathrm{j})}$ $=\frac{-1}{5}-\frac{1}{10}\mathrm{j}$	M1 A1 **[2]**	Attempt to multiply top and bottom by conjugate Or equivalent
8(ii)	$\lvert m\rvert=\sqrt{(-4)^2+2^2}=\sqrt{20}$ $\arg m=\pi-\arctan\left(\frac{1}{2}\right)=2.68$ So $m=\sqrt{20}\left(\cos 2.68+\mathrm{j}\sin 2.68\right)$	B1 M1 A1 A1(ft) **[4]**	Attempt to calculate angle Accept any correct expression for angle, including 153.4 degrees, –206 degrees and –3.61 (must be at least 3s.f.) Also accept $(r,\ \theta)$ form
8(iii) (*A*)		B1 B1 **[2]**	Correct initial point Half-line at correct angle
8(iii) (*B*)	Shaded region, excluding boundaries 	B1(ft) B1(ft) B1(ft) **[3]**	Correct horizontal half-line from starting point Correct region indicated Boundaries excluded (accept dotted lines)

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\mathbf{M}^{-1}=\frac{1}{3}\begin{pmatrix}1 & -2\\ 0 & 3\end{pmatrix}$ $\mathbf{N}^{-1}=\frac{1}{7}\begin{pmatrix}4 & 3\\ -1 & 1\end{pmatrix}$	M1 A1 A1 **[3]**	Dividing by determinant One for each inverse c.a.o.
9(ii)	$\mathbf{MN}=\begin{pmatrix}3 & 2\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & -3\\ 1 & 4\end{pmatrix}=\begin{pmatrix}5 & -1\\ 1 & 4\end{pmatrix}$	M1 A1	Must multiply in correct order
	$(\mathbf{MN})^{-1}=\frac{1}{21}\begin{pmatrix}4 & 1\\ -1 & 5\end{pmatrix}$	A1	Ft from **MN**
	$\mathbf{N}^{-1}\mathbf{M}^{-1}=\frac{1}{7}\begin{pmatrix}4 & 3\\ -1 & 1\end{pmatrix}\times\frac{1}{3}\begin{pmatrix}1 & -2\\ 0 & 3\end{pmatrix}$ $=\frac{1}{21}\begin{pmatrix}4 & 1\\ -1 & 5\end{pmatrix}$	M1 A1	Multiplication in correct order Ft from (i)
	$=(\mathbf{MN})^{-1}$	A1 **[6]**	Statement of equivalence to $(\mathbf{MN})^{-1}$
9(iii)	$\Rightarrow(\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1}=\mathbf{IQ}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}\mathbf{PI}=\mathbf{Q}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}\mathbf{P}=\mathbf{Q}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}\mathbf{PP}^{-1}=\mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}\mathbf{I}=\mathbf{Q}^{-1}\mathbf{P}^{-1}$ $\Rightarrow(\mathbf{PQ})^{-1}=\mathbf{Q}^{-1}\mathbf{P}^{-1}$	M1 M1 M1 A1 **[4]**	$\mathbf{QQ}^{-1}=\mathbf{I}$ Correctly eliminate **I** from LHS Post-multiply both sides by $\mathbf{P}^{-1}$ at an appropriate point Correct and complete argument
			Section B Total: 36
			Total: 72

4755 - Further Concepts for Advanced Mathematics (FP1)

General Comments

This paper was well answered with many high scores and few really low ones. However, the mistakes that some quite good candidates made left the impression that they may not have been quite ready to take the examination.

The paper was of an appropriate standard and length, with the high marks reflecting the talented candidature it attracts.

The entry continues to grow, indicating that centres are encouraging more candidates to study Further Mathematics.

Comments on Individual Questions

1 **Proof**

Almost all candidates got this question right.

2 **Roots of a quadratic**

While almost all candidates knew what was expected in this question, many lost marks through errors in simplifying $\frac{4 \pm \sqrt{-12}}{2}$.

The second part of the question asked for the points to be plotted on an Argand diagram and this was well answered.

3 **Matrix transformation**

In the first part of this question candidates applied a matrix to a triangle and plotted its image. This was well done, though several failed to label the image points clearly. Candidates were then asked to describe the transformation, which was a stretch with different scale factors in the *x*- and *y*- directions; many candidates said it was an enlargement and many more chose a combination of reflections and rotations, failing to take account of what had happened to the individual points.

4 **Series summation using standard results**

This question was well answered, though there was much evidence of careless notation. The most common mistake was failing to factorise the expression and some candidates multiplied everything out before attempting to factorise, which introduces many opportunities for error; another common mistake was to use the wrong standard results, typically $\sum r^2$ instead of $\sum r$.

5 **Manipulating the roots of an equation**

Although there were many correct answers to this question, it was also the case that many candidates who knew what they were doing lost marks through careless mistakes. A common mistake was to present the final answer as an expression rather than an equation. The question asked candidates to find a new equation with roots related to those of the given equation; the method of working from the sum and products of the roots of the given equation was more popular than that of substituting for *x* and was on the whole carried out slightly more accurately.

6 **Proof by induction**

This question was often well answered and full marks were fairly common. A few candidates failed to present the correct structure. Others were not explicit about the assumption that the result is true for $n = k$; statements like "Let $n = k$" were not uncommon. It was also not uncommon for candidates to skimp on the final few statements necessary to complete the proof and so lose marks.

Many candidates got into a muddle with notation, for example by writing $\sum(k+1)^2$ when they meant $\sum_{r=1}^{k+1} r^2$.

7 **Graph**

This question was well answered and even the weakest candidates were able to get some marks on it.

(i) This asked for the intercept with the *y*-axis. It was almost universally answered correctly.

(ii) This asked for the equations of the asymptotes. Almost all got the vertical asymptotes right, but a few failed to give the correct horizontal asymptote.

(iii) Most candidates sketched the graph correctly and labelled the asymptotes, but many lost a mark by failing to mark in the intercept with the *y*-axis.

(iv) Candidates were asked to solve a related inequality and this was less well answered; many gave only one of the three regions in which it held. They could have avoided this mistake had they used their graph sketch to help them.

8 **Complex numbers**

This question proved the most taxing for many candidates.

(i) Candidates were asked to find the reciprocal of the complex number $-4+2\mathrm{j}$. While most candidates got this right, many knew to multiply the numerator and denominator by the conjugate but then made careless mistakes; a few displayed complete ignorance of the topic, writing things like $\frac{1}{-4+2\mathrm{j}} = \frac{-1}{4} + \frac{1}{2}\mathrm{j}$.

(ii) Candidates were asked to write the complex number in modulus argument form; most could calculate the modulus, but had problems with the argument; many gave the supplement of the argument, a mistake that they would not have made if they had drawn a sketch Argand diagram to help them. Many calculated the modulus and argument but lost a mark by not actually expressing the complex number in modulus argument form, or by doing so incorrectly.

(iii) Part (iii) involved two related loci and many candidates lost some marks here, either by starting their half-lines at $-m$ rather than m, or by failing to draw the lines in part (iii) (B) broken to show they were excluded. A few candidates shaded the unwanted region but did not state that this was what they were doing. A few of the weaker candidates thought that the loci were circles.

Several candidates might have earned more marks had they ensured their diagrams were clearly labelled.

9 **Matrices**

Parts (i) and (ii) were generally done well. Part (iii) was more taxing but the standard of attempts was pleasing and many got full marks.

(i)(ii) In parts (i) and (ii) candidates were asked to multiply matrices and to find their inverses. This was well done and many candidates got full marks for these parts. Loss of marks was usually due to careless mistakes but some candidates were unaware that matrix multiplication is not commutative.

(iii) In part (iii) candidates were asked to prove a general result. Almost all candidates knew what was expected of them and many got it fully right; the commonest mistake was to fail to write the matrices on the left hand side in the correct order after post-multiplying by $\mathbf{P}^{-1}$.

The best solutions were annotated to set out very clearly the steps and properties involved.